Probabilistic Reasoning

Acting under Uncertainty

Definition: Acting under Uncertainty refers to making decisions in situations where outcomes are not completely predictable due to incomplete information or stochastic elements.

Key Concepts:

- **Uncertain Environments**: Includes situations where the consequences of actions are not fully known or are probabilistic.
- **Decision Making**: Involves strategies such as risk assessment, expected utility, and decision trees to optimize outcomes despite uncertainty.

Approaches:

- **Probabilistic Models**: Represent uncertainty using probabilities, enabling reasoning about likely outcomes and optimal decisions.
- **Decision Theory**: Provides frameworks like Expected Utility Theory to quantify preferences and make rational decisions under uncertainty.

Basic Probability Notation

Probability Basics:

- **Sample Space**: Set of all possible outcomes of an experiment.
- **Event**: Subset of the sample space, representing a set of outcomes.
- **Probability**: Measure of the likelihood of an event occurring, typically denoted as P(event).

Operations:

- **Union and Intersection**: $P(A \cup B)$ denotes the probability of either A or B occurring; $P(A \cap B)$ denotes the probability of both A and B occurring.
- **Complement**: $P(A')$ denotes the probability of the complement of event A (not A).

Conditional Probability:

- **Definition**: $P(A \mid B)$ denotes the probability of event A occurring given that event B has occurred.
- **Formula**: $P(A \mid B) = P(A \cap B) / P(B)$, where $P(B) \neq 0$.

Bayes' Rule and Its Use

Bayes' Rule:

- **Formula**: P(A | B) = P(B | A) * P(A) / P(B), where:
  - P(A | B) is the probability of A given B.
  - P(B | A) is the probability of B given A.
  - P(A) and P(B) are the probabilities of A and B respectively.

Applications:

- **Bayesian Inference**: Updates prior beliefs (P(A)) based on new evidence (P(B | A)) to calculate posterior probabilities (P(A | B)).
- **Diagnostic Reasoning**: Calculates probabilities of diseases given symptoms, incorporating prior knowledge and observed data.

Use Cases:

- **Machine Learning**: Bayesian networks use Bayes' Rule for probabilistic graphical models, aiding in decision-making under uncertainty.
- **Medical Diagnosis**: Determines disease probabilities based on symptoms and patient history.

Probabilistic Reasoning

Definition: Probabilistic Reasoning involves making decisions and predictions under uncertainty using probability theory.

Key Concepts:

- **Uncertainty Types**: Includes stochastic outcomes, incomplete information, and ambiguity in decision-making.
- **Probabilistic Models**: Represent uncertainty using probabilities, enabling quantitative reasoning about likely outcomes.

Approaches:

- **Bayesian Networks**: Graphical models that encode probabilistic relationships between variables.
- **Decision Theory**: Frameworks like Expected Utility Theory to optimize decisions given uncertain outcomes.

Applications:

- **AI Planning**: Incorporates uncertainty into plans and strategies.

- **Medical Diagnosis**: Calculates probabilities of diseases based on symptoms and patient history.

Representing Knowledge in an Uncertain Domain

Challenges:

- **Incomplete Information**: Missing or uncertain data affecting decision-making.
- **Stochastic Processes**: Random variables influencing outcomes.

Techniques:

- **Probability Distributions**: Model uncertainty using distributions like Gaussian, Bernoulli, or Poisson.
- **Belief Networks**: Represent conditional dependencies between variables using directed acyclic graphs (DAGs).

Example:

- **Weather Prediction**: Using historical data and probabilistic models to forecast future weather conditions.

The Semantics of Bayesian Networks

Bayesian Networks (BN):

- **Definition**: Graphical representation of probabilistic relationships between variables, based on Bayes' Rule.
- **Components**:
  - **Nodes**: Represent variables, each with a probability distribution.
  - **Edges**: Directed edges denote probabilistic dependencies between variables.
  - **Conditional Probabilities**: Quantify how one variable influences another.

Semantics:

- **Conditional Independence**: Nodes are conditionally independent of their non-descendants given their parents.
- **Propagation**: Update beliefs (probabilities) using evidence and probabilistic inference algorithms (like Pearl's Belief Propagation).

Use Cases:

- **Medical Diagnosis**: Bayesian networks aid in diagnosing diseases based on symptoms and test results.

- **Risk Assessment**: Evaluate probabilities of risks and mitigating factors in decision-making scenarios.

Efficient Representation of Conditional Distributions

Challenges:

- **Complexity**: Large state spaces and dependencies increase computational complexity.
- **Storage**: Efficiently storing and accessing conditional probabilities.

Techniques:

- **Parameterization**: Represent distributions using parameters (e.g., mean and variance for Gaussian distributions).
- **Factorization**: Decompose joint distributions into smaller, manageable factors (e.g., using factor graphs).

Methods:

- **Conditional Independence**: Exploit conditional independence assumptions to simplify distributions.
- **Sparse Representations**: Use sparse matrices or data structures to store conditional probabilities efficiently.

Approximate Inference in Bayesian Networks

Problem:

- **Intractability**: Exact inference in large Bayesian networks is computationally expensive.
- **Approximate Methods**: Seek to balance accuracy and computational feasibility.

Approaches:

- **Sampling Methods**: Use Monte Carlo techniques (e.g., Markov Chain Monte Carlo) to approximate posterior distributions.
- **Variational Inference**: Approximate complex posterior distributions with simpler distributions, optimizing a divergence measure.

Trade-offs:

- **Accuracy vs. Speed**: Approximate methods sacrifice accuracy for faster computation.
- **Convergence**: Ensure algorithms converge to valid solutions despite approximations.

Relational and First-Order Probability

Definition:

- **Relational Probability**: Extends probability theory to handle uncertainty in relational data and dependencies between entities.
- **First-Order Probability**: Integrates logical and probabilistic reasoning, combining predicate logic with probability distributions.

Applications:

- **Knowledge Representation**: Model complex relationships and dependencies in relational databases.
- **Uncertain Reasoning**: Infer uncertain outcomes based on relational data patterns and logical rules.

Methods:

- **Probabilistic Relational Models (PRMs)**: Extend Bayesian networks to relational domains, capturing dependencies between entities.
- **Markov Logic Networks (MLNs)**: Integrate logic and probability, allowing for probabilistic reasoning over relational structures.